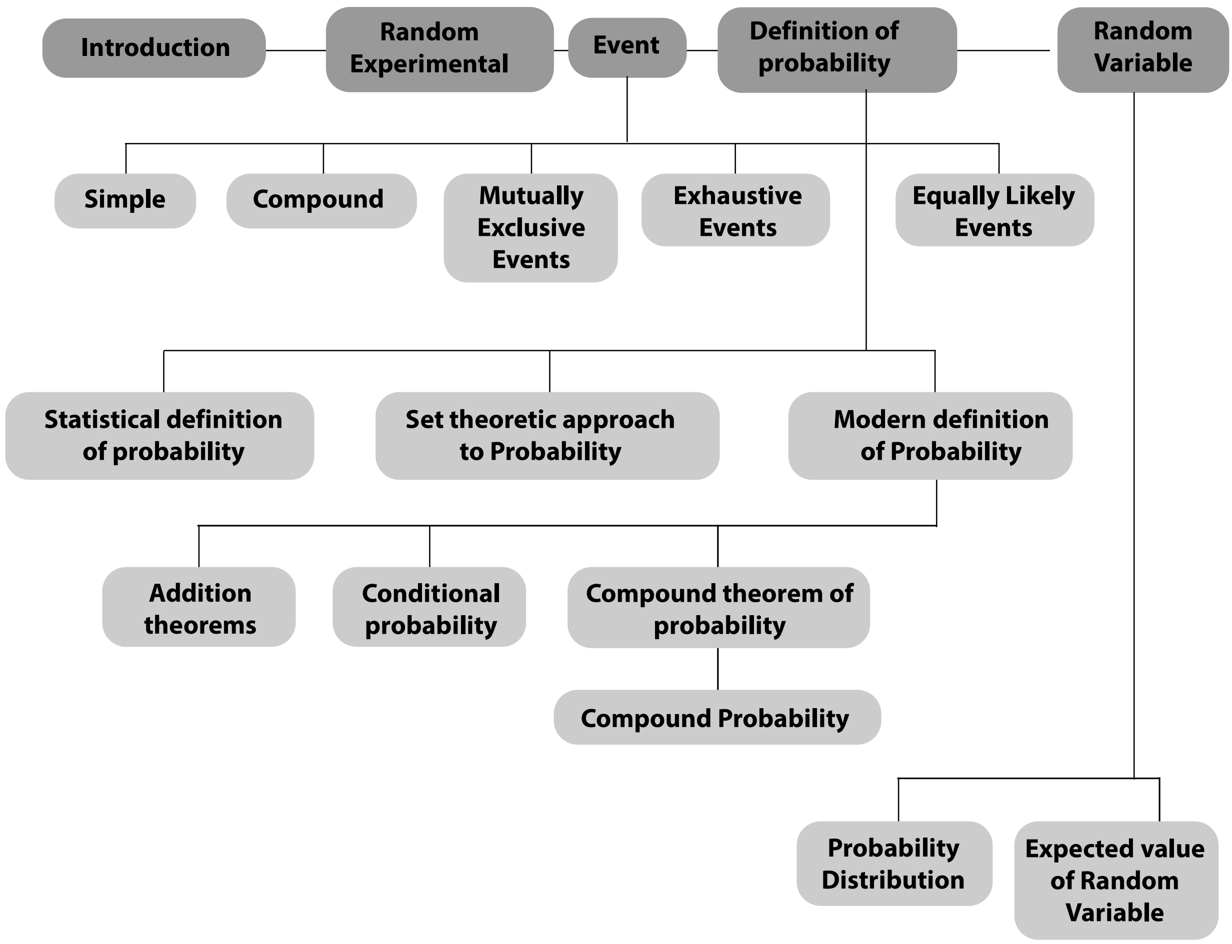
CHAPTER 15

PROBABILITY

LEARNING OBJECTIVES

Concept of probability is used in accounting and finance to understand the likelihood of occurrence or non-occurrence of a variable. It helps in developing financial forecasting in which you need to develop expertise at an advanced stage of chartered accountancy course.

CHAPTER OVERVIEW



15.1 INTRODUCTION

The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability. In recent time, probability has developed itself into a full-fledged subject and become an integral part of statistics. The theories of Testing Hypothesis and Estimation are based on probability.

It is rather surprising to know that the first application of probability was made by a group of mathematicians in Europe about three hundreds years back to enhance their chances of winning in different games of gambling. Later on, the theory of probability was developed by Abraham De Moicere and Piere-Simon De Laplace of France, Reverend Thomas Bayes and R. A. Fisher of England, Chebyshev, Morkov, Khinchin, Kolmogorov of Russia and many other noted mathematicians as well as statisticians.

Two broad divisions of probability are Subjective Probability and Objective Probability. Subjective Probability is basically dependent on personal judgement and experience and, as such, it may be influenced by the personal belief, attitude and bias of the person applying it. However in the field of uncertainty, this would be quite helpful and it is being applied in the area of decision making management. This Subjective Probability is beyond the scope of our present discussion. We are going to discuss Objective Probability in the remaining sections.

15.2 RANDOM EXPERIMENT

In order to develop a sound knowledge about probability, it is necessary to get ourselves familiar with a few terms.

Experiment: An experiment may be described as a performance that produces certain results.

Random Experiment: An experiment is defined to be random if the results of the experiment depend on chance only. For example if a coin is tossed, then we get two outcomes—Head (H) and Tail (T). It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once. Thus, tossing a coin is an example of a random experiment. Similarly, rolling a dice (or any number of dice), drawing items from a box containing both defective and non—defective items, drawing cards from a pack of well shuffled fifty two cards etc. are all random experiments.

Events: The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types:

(i) Simple or Elementary,

(ii) Composite or Compound.

An event is known to be simple if it cannot be decomposed into further events. Tossing a coin once provides us two simple events namely Head and Tail. On the other hand, a composite event is one that can be decomposed into two or more events. Getting a head when a coin is tossed twice is an example of composite event as it can be split into the events HT and TH which are both elementary events.

Mutually Exclusive Events or Incompatible Events: A set of events A1, A2, A3, …… is known to be mutually exclusive if not more than one of them can occur simultaneously. Thus occurrence of one such event implies the non-occurrence of the other events of the set. Once a coin is tossed, we get two mutually exclusive events Head and Tail.

Exhaustive Events: The events A1, A2, A3, ………… are known to form an exhaustive set if one of these events must necessarily occur. As an example, the two events Head and Tail, when a coin is tossed once, are exhaustive as no other event except these two can occur.

Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events: The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events. The two events Head and Tail when a coin is tossed is an example of a pair of equally likely events because there is no reason to assume that Head (or Tail) would occur more frequently as compared to Tail (or Head).

15.3 CLASSICAL DEFINITION OF PROBABILITY OR A PRIORI DEFINITION

Let us consider a random experiment that result in n finite elementary events, which are assumed to be equally likely. We next assume that out of these n events, n_A $(\leq n)$ events are favourable to an event A. Then the probability of occurrence of the event A is defined as the ratio of the number of events favourable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{n_A}{n} = \frac{\text{No. of equally likely events favourable to A}}{\text{Total no. of equally likely events}} \quad \ldots\ldots\ldots\ldots\ldots\ (15.1)$$

However if instead of considering all elementary events, we focus our attention to only those composite events, which are mutually exclusive, exhaustive and equally likely and if $m(\leq n)$ denotes such events and is furthermore $m_A(\leq n_A)$ denotes the no. of mutually exclusive, exhaustive and equally likely events favourable to A, then we have

$$P(A) = \frac{m_A}{m} = \frac{\text{No. of mutually exclusive, exhaustive and equally likely events favourable to A}}{\text{Total no. of mutually exclusive, exhaustive and equally likely events}}$$

……………… (15.2)

For this definition of probability, we are indebted to Bernoulli and Laplace. This definition is also termed as a priori definition because probability of the event A is defined on the basis of prior knowledge.

This classical definition of probability has the following demerits or limitations:

(i) It is applicable only when the total no. of events is finite.

(ii) It can be used only when the events are equally likely or equi-probable. This assumption is made well before the experiment is performed.

(iii) This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

In connection with classical definition of probability, we may note the following points:

(a) The probability of an event lies between 0 and 1, both inclusive.

i.e. $0 \leq P(A) \leq 1$ .... (15.3)

When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.

(b) Non-occurrence of event A is denoted by A′ or A^C or $\bar{A}$ and it is known as complimentary event of A. The event A along with its complimentary A′ forms a set of mutually exclusive and exhaustive events.

i.e. $P(A) + P(A') = 1$

$$\Rightarrow \quad P(A') = 1 - P(A)$$

$$1 - \frac{m_A}{m}$$

$$= \frac{m - m_A}{m} \qquad \text{...............} (15.4)$$

(c) The ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event A and its inverse ratio is known as odds against the event A.

i.e. odds in favour of A $= m_A : (m - m_A)$ .... (15.5)

and odds against A $= (m - m_A) : m_A$ .... (15.6)

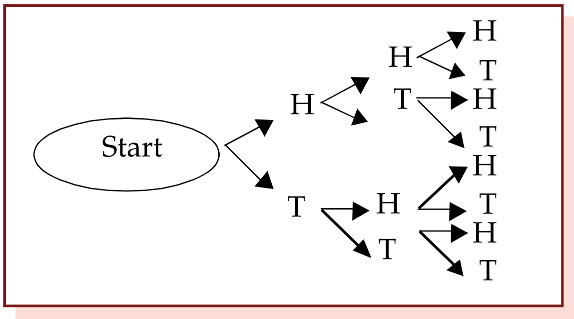
ILLUSTRATIONS:

Example 15.1: A coin is tossed three times. What is the probability of getting:

(i) 2 heads

(ii) at least 2 heads.

Solution: When a coin is tossed three times, first we need enumerate all the elementary events. This can be done using 'Tree diagram' as shown below:



Hence the elementary events are

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Thus the number of elementary events (n) is 8.

(i) Out of these 8 outcomes, 2 heads occur in three cases namely HHT, HTH and THH. If we denote the occurrence of 2 heads by the event A and if assume that the coin as well as performer of the experiment is unbiased then this assumption ensures that all the eight elementary events are equally likely. Then by the classical definition of probability, we have

$$P(A) = \frac{n_A}{n} = \frac{3}{8} = 0.375$$

(ii) Let B denote occurrence of at least 2 heads i.e. 2 heads or 3 heads. Since 2 heads occur in 3 cases and 3 heads occur in only 1 case, B occurs in 3 + 1 or 4 cases. By the classical definition of probability,

$$P(B) = \frac{4}{8} = 0.50$$

Example 15.2: A dice is rolled twice. What is the probability of getting a difference of 2 points?

Solution: If an experiment results in p outcomes and if the experiment is repeated q times, then the total number of outcomes is p^q. In the present case, since a dice results in 6 outcomes and the dice is rolled twice, total no. of outcomes or elementary events is 6^2 or 36. We assume that the dice is unbiased which ensures that all these 36 elementary events are equally likely. Now a difference of 2 points in the uppermost faces of the dice thrown twice can occur in the following cases:

1st Throw	2nd Throw	Difference
6	4	2
5	3	2
4	2	2
3	1	2
1	3	2
2	4	2
3	5	2
4	6	2

Thus denoting the event of getting a difference of 2 points by A, we find that the no. of outcomes favourable to A, from the above table, is 8. By classical definition of probability, we get

$$P(A) = \frac{8}{36}$$

$$= \frac{2}{9}$$

Example 15.3: Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.

Solution: If two dice are thrown then, as explained in the last problem, total no. of elementary events is 6^2 or 36. Now a total of 7 or more i.e. 7 or 8 or 9 or 10 or 11 or 12 can occur only in the following combinations:

SUM = 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

SUM = 8: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

SUM = 9: (3, 6), (4, 5), (5, 4), (6, 3)

SUM = 10: (4, 6), (5, 5), (6, 4)

SUM = 11: (5, 6), (6, 5)

SUM = 12: (6, 6)

Thus the no. of favourable outcomes is 21. Letting A stand for getting a total of 7 points or more, we have

$$P(A) = \frac{21}{36}$$

$$= \frac{7}{12}$$

Example 15.4: What is the chance of picking a spade or an ace not of spade from a pack of 52 cards?

Solution: A pack of 52 cards contain 13 Spades, 13 Hearts, 13 Clubs and 13 Diamonds. Each of these groups of 13 cards has an ace. Hence the total number of elementary events is 52 out of which 13 + 3 or 16 are favourable to the event A representing picking a Spade or an ace not of Spade. Thus we have

$$P(A) = \frac{16}{52} = \frac{4}{13}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} = \frac{3}{52} - 0 = \frac{16}{52} = \frac{4}{13}$$

Example 15.5: Find the probability that a four digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.

Solution: Since there are four digits, all distinct, the total number of four digit numbers that can be formed without any restriction is 4! or $4 \times 3 \times 2 \times 1$ or 24. Now a four digit number would be divisible by 4 if the number formed by the last two digits is divisible by 4. This could happen when the four digit number ends with 52 or 56 or 72 or 76. If we fix the last two digits by 52, and then the 1st two places of the four digit number can be filled up using the remaining 2 digits in 2! or 2 ways. Thus there are 2 four digit numbers that end with 52. Proceeding in this manner, we find that the number of four digit numbers that are divisible by 4 is 4×2 or 8. If (A) denotes the event that any four digit number using the given digits would be divisible by 4, then we have

$$P(A) = \frac{8}{24}$$

$$= \frac{1}{3}$$

Example 15.6: A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise:

(a) 2 ladies,

(b) at least 2 ladies.

Solution: Since there are altogether 8 + 5 or 13 persons, a committee comprising 7 members can be formed in

$$^{13}C_7 \quad \text{or} \quad \frac{13!}{7!6!} \quad \text{or} \quad \frac{13\times12\times11\times10\times9\times8\times7!}{7!\times6\times5\times4\times3\times2\times1}$$

or $11 \times 12 \times 13$ ways.

(a) When the committee is formed taking 2 ladies out of 5 ladies, the remaining (7–2) or 5 committee members are to be selected from 8 gentlemen. Now 2 out of 5 ladies can be selected in $^5C_2$ ways and 5 out of 8 gentlemen can be selected in $^8C_5$ ways. Thus if A denotes the event of having the committee with 2 ladies, then A can occur in $^5C_2 \times {}^8C_5$ or

$$\frac{5\times4}{2\times1}\times\frac{8\times7\times6}{3\times2} \text{ or } 10 \times 56 \text{ ways.}$$

Thus $$P(A) = \frac{10\times56}{11\times12\times13}$$

$$= \frac{140}{429}$$

(b) Since the minimum number of ladies is 2, we can have the following combinations:

Population:	5L		8G
Sample:	2L	+	5G
or	3L	+	4G
or	4L	+	3G
or	5L	+	2G

Thus if B denotes the event of having at least two ladies in the committee, then B can occur in

$^5C_2 \times {}^8C_5 + {}^5C_3 \times {}^8C_4 + {}^5C_4 \times {}^8C_3 + {}^5C_5 \times {}^8C_2$

i.e. 1568 ways.

Hence $$P(B) = \frac{1568}{11\times12\times13}$$

$$= \frac{392}{429}$$

15.4 RELATIVE FREQUENCY DEFINITION OF PROBABILITY

Owing to the limitations of the classical definition of probability, there are cases when we consider the statistical definition of probability based on the concept of relative frequency. This definition of probability was first developed by the British mathematicians in connection with the survival probability of a group of people.

Let us consider a random experiment repeated a very good number of times, say n, under an identical set of conditions. We next assume that an event A occurs f_A times. Then the limiting value of the ratio of f_A to n as n tends to infinity is defined as the probability of A.

i.e. $$P(A) = \lim_{n \to \infty} \frac{F_A}{n} \qquad \text{.................. (15.7)}$$

This statistical definition is applicable if the above limit exists and tends to a finite value.

Example 15.7: The following data relate to the distribution of wages of a group of workers:

Wages in ₹:	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers:	15	23	36	42	17	12	5

If a worker is selected at random from the entire group of workers, what is the probability that

(a) his wage would be less than ₹ 50?

(b) his wage would be less than ₹ 80?

(c) his wage would be more than ₹ 100?

(d) his wages would be between ₹ 70 and ₹ 100?

Solution: As there are altogether 150 workers, n = 150.

(a) Since there is no worker with wage less than ₹ 50, the probability that the wage of a randomly selected worker would be less than ₹ 50 is $P(A) = \frac{0}{150} = 0$

(b) Since there are (15+23+36) or 74 worker having wages less than ₹ 80 out of a group of 150 workers, the probability that the wage of a worker, selected at random from the group, would be less than ₹ 80 is

$$P(B) = \frac{74}{150} = \frac{37}{75}$$

(c) There are (12+5) or 17 workers with wages more than ₹ 100. Thus the probability of finding a worker, selected at random, with wage more than ₹ 100 is

$$P(C) = \frac{17}{150}$$

(d) There are (36+42+17) or 95 workers with wages in between ₹ 70 and ₹ 100. Thus

$$P(D) = \frac{95}{150} = \frac{19}{30}$$

15.5 OPERATIONS ON EVENTS-SET THEORETIC APPROACH TO PROBABILITY

Applying the concept of set theory, we can give a new dimension to the classical definition of probability. A sample space may be defined as a non-empty set containing all the elementary events of a random experiment as sample points. A sample space is denoted by S or Ω. An event A may be defined as a non-empty subset of S. This is shown in Figure 15.1

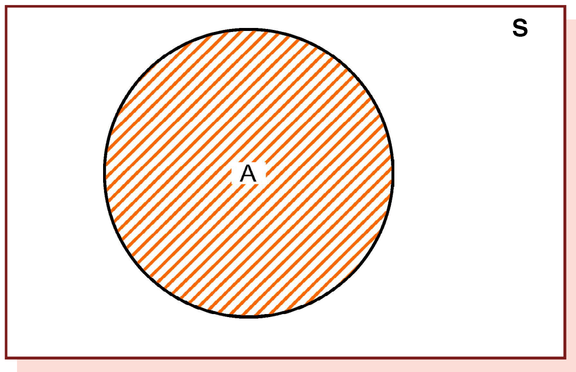


Figure 15.1

Showing an event A and the sample space S

As for example, if a dice is rolled once than the sample space is given by

S = {1, 2, 3, 4, 5, 6}.

Next, if we define the events A, B and C such that

A = {x: x is an even no. of points in S}

B = {x: x is an odd no. of points in S}

C = {x: x is a multiple of 3 points in S}

Then, it is quite obvious that

A = {2, 4, 6}, B = {1, 3, 5} and C = {3, 6}.

The classical definition of probability may be defined in the following way.

Let us consider a finite sample space S i.e. a sample space with a finite no. of sample points, n (S). We assume that all these sample points are equally likely. If an event A which is a subset of S, contains n (A) sample points, then the probability of A is defined as the ratio of the number of sample points in A to the total number of sample points in S. i.e.

$$P(A) = \frac{n(A)}{n(S)} \qquad \text{.....................} \quad (15.8)$$

Union and Intersection of Two Events

Union of two events A and B is defined as a set of events containing all the sample points of event A or event B or both the events. This is shown in Figure 15.2 and

we have $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

where x denotes the sample points.

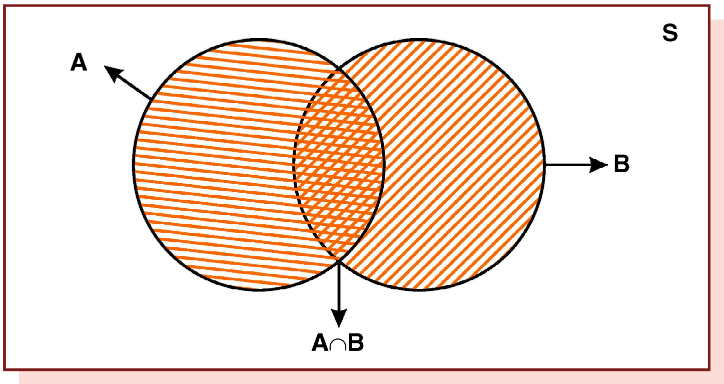


Figure 15.2

Showing the union of two events A and B and also their intersection

In the above example, we have $A \cup C = \{2, 3, 4, 6\}$

and $A \cup B = \{1, 2, 3, 4, 5, 6\}$.

The intersection of two events A and B may be defined as the set containing all the sample points that are common to both the events A and B. This is shown in Figure 15.2. we have

$A \cap B = \{x : x \in A \text{ and } x \in B\}$.

In the above example, $A \cap B = \phi$

$A \cap C = \{6\}$

Since the intersection of the events A and B is a null set ϕ, it is obvious that A and B are mutually exclusive events as they cannot occur simultaneously.

The difference of two events A and B, to be denoted by A – B, may be defined as the set of sample points present in set A but not in B. i.e.

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, B – A = $\{x : x \in B$ and $x \notin A\}$.

In the above examples,

A – B = ϕ

And A – C = {2, 4}.

This is shown in Figure 15.3.

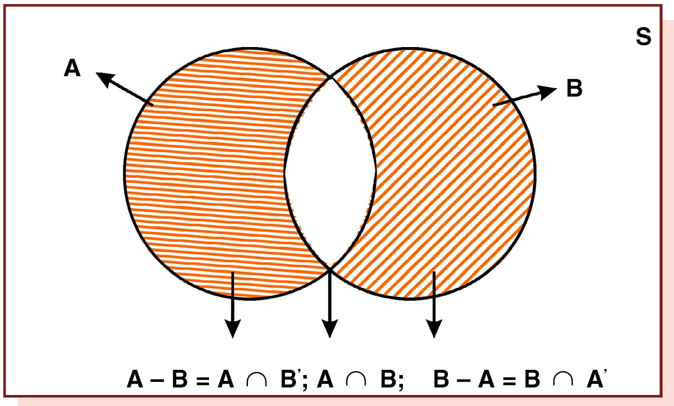


Figure 15.3

Showing (A – B) and (B – A)

The complement of an event A may be defined as the difference between the sample space S and the event A. i.e.

$A' = \{x : x \in S$ and $x \notin A\}$.

In the above example A′= S – A

= {1, 3, 5}

Figure 15.4 depicts A′

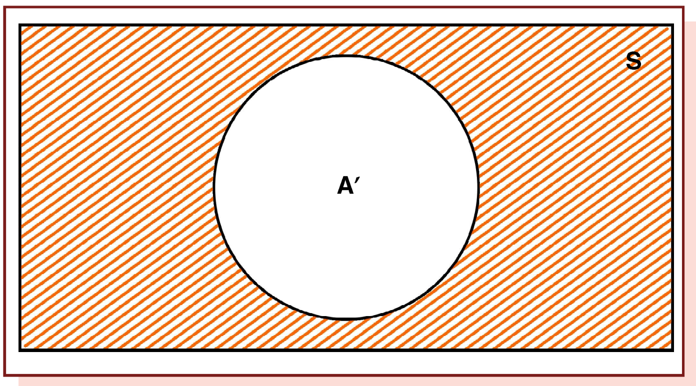


Figure 15.4

Showing A′

Now we are in a position to redefine some of the terms we have already discussed in this section.

Two events A and B are mutually exclusive if $P(A \cap B) = 0$ or more precisely, ...(15.9)

$P(A \cup B) = P(A) + P(B)$ ....(15.10)

Similarly three events A, B and C are mutually exclusive if

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$ ....(15.11)

Two events A and B are exhaustive if

$P(A \cup B) = 1$ ....(15.12)

Similarly three events A, B and C are exhaustive if

$P(A \cup B \cup C) = 1$ .... (15.13)

Three events A, B and C are equally likely if

$P(A) = P(B) = P(C)$ .... (15.14)

Example 15.8: Three events A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of A?

Solution: Since A, B and C are mutually exclusive, we have

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$ .... (1)

Since they are exhaustive, $P(A \cup B \cup C) = 1$ .... (2)

Since they are also equally likely, $P(A) = P(B) = P(C) = K$, Say ....(3)

Combining equations (1), (2) and (3), we have

$1 = K + K + K$

$\Rightarrow K = 1/3$

Thus $P(A) = P(B) = P(C) = 1/3$

Hence $P(A') = 1 - 1/3 = 2/3$

15.6 AXIOMATIC OR MODERN DEFINITION OF PROBABILITY

Let us consider a sample space S in connection with a random experiment and let A be an event defined on the sample space S i.e. $A \subseteq S$. Then a real valued function P defined on S is known as a probability measure and P(A) is defined as the probability of A if P satisfies the following axioms:

(i) $P(A) \geq 0$ for every $A \subseteq S$ (subset) .... (15.15)

(ii) $P(S) = 1$ .... (15.16)

(iii) For any sequence of mutually exclusive events $A_1, A_2, A_3,..$

$P(A_1 \cup A_2 \cup A_3 \cup ....) = P(A_1) + P(A_2) + P(A_3) +$ .... (15.17)

15.7 ADDITION THEOREMS OR THEOREMS ON TOTAL PROBABILITY

Theorem 1 For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.

i.e. $P(A \cup B)$

or $P(A + B) = P(A) + P(B)$ .... (15.18)

or $P(A \text{ or } B)$ whenever A and B are mutually exclusive

This is illustrated in the following example.

Example 15.9: A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?

Solution: Let A be the event that the number selected would be divisible by 4 and B, the event that the selected number would be divisible by 7. Then AUB denotes the event that the number would be divisible by 4 or 7. Next we note that A = {4, 8, 12, 16, 20, 24} and B = {7, 14, 21} whereas S = {1, 2, 3, ……... 25}. Since $A \cap B = \phi$, the two events A and B are mutually exclusive and as such we have

$P(A \cup B) = P(A) + P(A)$ .... (1)

Since $P(A) = \frac{n(A)}{n(S)} = \frac{6}{25}$

and $P(B) = \frac{n(B)}{n(S)} = \frac{3}{25}$

Thus from (1), we have

$$P(A \cup B) = \frac{6}{25} + \frac{3}{25}$$

$$= \frac{9}{25}$$

Hence the probability that the selected number would be divisible by 4 or 7 is 9/25 or 0.36

Example 15.10: A coin is tossed thrice. What is the probability of getting 2 or more heads?

Solution: If a coin is tossed three times, then we have the following sample space.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} 2 or more heads imply 2 or 3 heads.

If A and B denote the events of occurrence of 2 and 3 heads respectively, then we find that

A = {HHT, HTH, THH} and B = {HHH}

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

and $P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$

As A and B are mutually exclusive, the probability of getting 2 or more heads is

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{8} + \frac{1}{8}$$

$$= 0.50$$

Theorem 2 For any $K(\geq 2)$ mutually exclusive events $A_1, A_2, A_3 \ldots, A_K$ the probability that at least one of them occurs is given by the sum of the individual probabilities of the K events.

i.e. $P(A_1 \cup A_2 \cup \ldots \cup A_K) = P(A_1) + P(A_2) + \ldots. P(A_K)$ .... (15.19)

Obviously, this is an extension of Theorem 1.

Theorem 3 For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

i. e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .... (15.20)

This theorem is stronger than Theorem 1 as we can derive Theorem 1 from Theorem 3 and not Theorem 3 from Theorem 1. For want of sufficient evidence, it is wiser to apply Theorem 3 for evaluating total probability of two events.

Example 15.11: A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9?

Solution: Let A, B, $A \cup B$ and $A \cap B$ denote the events that the selected number would be a multiple of 5, 9, 5 or 9 and both 5 and 9 i.e. LCM of 5 and 9 i.e. 45 respectively.

multiple of 5 since $1000 = 5 \times 200$

multiple of $9 = 9 \times 111 + 1$

Both 5 and $9 = 45 \times 22 + 10$,

it is obvious that

$$P(A) = \frac{200}{1000}, P(B) = \frac{111}{1000}, P(A \cap B) = \frac{22}{1000}$$

Hence the probability that the selected number would be a multiple of 4 or 9 is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{200}{1000} + \frac{111}{1000} - \frac{22}{1000}$$

$$= 0.29$$

Example 15.12: The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

Solution: Let the event that the applicant is a B. Com. be denoted by B and that he is a CA be denoted by C Then as given,

$P(B) = 0.85$, $P(C) = 0.30$ and $P(B \cap C) = 0.25$

The probability that an applicant is B. Com. or CA is given by

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= 0.85 + 0.30 - 0.25$

$= 0.90$

Expected frequency $= N \times P(B \cup C)$

Expected frequency $= 500 \times 0.90 = 450$

Example 15.13: If $P(A-B) = 1/5$, $P(A) = 1/3$ and $P(B) = 1/2$, what is the probability that out of the two events A and B, only B would occur?

Solution: A glance at Figure 15.3 suggests that

$P(A-B) = P(A \cap B') = P(A) - P(A \cap B)$ …………..(15.21)

And $P(B-A) = P(B \cap A') = P(B) - P(A \cap B)$ …………..(15.22)

Also (15.21) and (15.22) describe the probabilities of occurrence of the event only A and only B respectively.

As given $P(A-B) = \frac{1}{5}$

$\Rightarrow \quad P(A) - P(A \cap B) = \frac{1}{5}$

$\Rightarrow \quad \frac{1}{3} - P(A \cap B) = \frac{1}{5}$ [Since $P(A) = 1/3$]

$\Rightarrow \quad P(A \cap B) = \frac{2}{15}$

The probability that the event B only would occur

$= P(B-A)$

$= P(B) - P(A \cap B)$

$= \frac{1}{2} - \frac{2}{15}$ [Since $P(B) = \frac{1}{2}$]

$= \frac{11}{30}$

Theorem 4 For any three events A, B and C, the probability that at least one of the events occurs is given by

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad \text{..........} (15.23)$$

Following is an application of this theorem.

Example 15.14: There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

Solution: As given $P(A) = 0.80$, $P(B) = 0.60$, $P(C) = 0.50$,

$P(A \cap B) = 0.46$, $P(B \cap C) = 0.32$, $P(A \cap C) = 0.48$ and

$P(A \cap B \cap C) = 0.26$

The probability that at least one of them survives another 5 years in given by

$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$ .... (15.23)

$= 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$

$= 0.90$

Expected Frequency = N X $P(A \cup B \cup C)$
=500 X 0.90= 450

15.8 CONDITIONAL PROBABILITY AND COMPOUND THEOREM OF PROBABILITY

Compound Probability or Joint Probability

The probability of an event, discussed so far, is technically known as unconditional or marginal probability. However, there are situations that demand the probability of occurrence of more than one event. The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by $P(A \cap B)$. In a similar manner, the probability of simultaneous occurrence of K events $A_1, A_2, \ldots A_k$, is denoted by $P(A_1 \cap A_2 \cap \ldots \cap A_k)$.

In case of compound probability of 2 events A and B, we may face two different situations. In the first case, if the occurrence of one event, say B, is influenced by the occurrence of another event A, then the two events A and B are known as dependent events. We use the notation P(B/A), to be read as 'probability of the event B given that the event A has already occurred (or 'the conditional probability of B given A) to suggest that another event B will happen if and only if the first event A has already happened. This is given by

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} \qquad \ldots\ldots\ldots (15.24)$$

Provided P(A) > 0 i.e. A is not an impossible event.

Similarly, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ ………… (15.25)

if P(B) > 0.

As an example if a box contains 5 red and 8 white balls and two successive draws of 2 balls are made from it without replacement then the probability of the event 'the second draw would result in 2 white balls given that the first draw has resulted in 2 Red balls' is an example of conditional probability since the drawings are made without replacement, the composition of the balls in the box changes and the occurrence of 2 white balls in the second draw (B_2) is dependent on the outcome of the first draw (R_2). This event may b denoted by

$P(B_2/R_2)$.

In the second scenario, if the occurrence of the second event B is not influenced by the occurrence of the first event A, then B is known to be independent of A. It also follows that in this case, A is also independent of B and A and B are known as mutually independent or just independent. In this case, we have

$P(B/A) = P(B)$ ………. (15.26)

and also $P(A/B) = P(A)$ ………. (15.27)

There by implying, $P(A \cap B) = P(A) \times P(B)$ ………. (15.28)

[From (15.24) or (15.25)]

In the above example, if the balls are drawn with replacement, then the two events B_2 and R_2 are independent and we have

$P(B_2 / R_2) = P(B_2)$

(15.28) is the necessary and sufficient condition for the independence of two events. In a similar manner, three events A, B and C are known as independent if the following conditions hold :

$P(A \cap B) = P(A) \times P(B)$

$P(A \cap C) = P(A) \times P(C)$

$P(B \cap C) = P(B) \times P(C)$

$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$ ……… (15.29)

It may be further noted that if two events A and B are independent, then the following pairs of events are also independent:

(i) A and B′

(ii) A′ and B

(iii) A′ and B′ ……… (15.30)

Theorems of Compound Probability

Theorem 5 For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred

i.e. $P(A \cap B) = P(A) \times P(B/A)$ Provided $P(A) > 0$ ……………(15.31)

Theorem 6 For any three events A, B and C, the probability that they occur jointly is given by

$P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B))$ Provided $P(A \cap B) > 0$ … (15.32)

In the event of independence of the events

(15.31) and (15.32) are reduced to

$P(A \cap B) = P(A) \times P(B)$

and $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

which we have already discussed.

Example 15.15: Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?

Solution: Let A denote the event that Rupesh hits the target and B, the event that David hits the target. Then as given,

$$P(A) = \frac{5}{9},\ P(B) = \frac{6}{11}$$

and $P(A \cap B) = P(A) \times P(B)$

$$= \frac{5}{9} \times \frac{6}{11}$$

$$= \frac{10}{33} \text{ (as A and B are independent)}$$

The probability that the target would be hit is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{9} + \frac{6}{11} - \frac{10}{33}$$

$$= \frac{79}{99}$$

Alternately $P(A \cup B) = 1 - P(A \cup B)'$

$= 1 - P(A' \cap B')$(by De-Morgan's Law)

$= 1 - P(A') \times P(B')$

$$= 1 - [1 - P(A)] \times [1 - P(B)] \qquad \text{(by 15.30)}$$

$$= 1 - \left(1 - \frac{5}{9}\right) \times \left(1 - \frac{6}{11}\right)$$

$$= 1 - \left(\frac{4}{9} \times \frac{5}{11}\right)$$

$$= \frac{79}{99}$$

Example 15.16: A pair of dice is thrown together and the sum of points of the two dice is noted to be 10. What is the probability that one of the two dice has shown the point 4?

Solution: Let A denote the event of getting 4 points on one of the two dice and B denote the event of getting a total of 10 points on the two dice. Then we have

$$P(A) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$\text{and } P(A \cap B) = \frac{2}{36}$$

[Since a total of 10 points may result in (4, 6) or (5, 5) or (6, 4) and two of these combinations contain 4]

$$\text{Thus } P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{2/36}{1/12}$$

$$= \frac{2}{3}$$

Alternately The sample space for getting a total of 10 points when two dice are thrown simultaneously is given by

$$S = \{(4, 6), (5, 5), (6, 4)\}$$

Out of these 3 cases, we get 4 in 2 cases. Thus by the definition of probability, we have

$$P(B/A) = \frac{2}{3}$$

Example 15.17: In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?

Solution: Let S and M stand for service holder and male respectively. We are to evaluate P (S / M).

We note that $(S \cap M)$ represents the event of both service holder and male.

Thus $P(S/M) = \dfrac{P(S \cap M)}{P(M)}$

$= \dfrac{12/35}{20/35}$

$= 0.60$

Example 15.18: In connection with a random experiment, it is found that

$P(A) = \dfrac{2}{3}, P(B) \dfrac{3}{5} =$ and $P(A \cup B) = \dfrac{5}{6}$

Evaluate the following probabilities:

(i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A'/B)$ (iv) $P(A/B')$ (v) $P(A'/B')$

Solution: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$=> \frac{5}{6} = \frac{2}{3} + \frac{3}{5} - P(A \cap B)$$

$$=> P(A \cap B) = \frac{2}{3} + \frac{3}{5} - \frac{5}{6}$$

$$= \frac{13}{30}$$

Hence (i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{13/30}{3/5} = \dfrac{13}{18}$

(ii) $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{13/30}{2/3} = \dfrac{13}{20}$

(iii) $P(A'/B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{\frac{3}{5} - \frac{13}{30}}{\frac{3}{5}} = \dfrac{5}{18}$

(iv) $(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)} = \dfrac{7}{12}$

(v) $P(A'/B') = \dfrac{P(A' \cap B')}{P(B')}$

$$= \frac{P(A \cup B)'}{P(B')} \quad [\text{ by De-Morgan's Law } A' \cap B' = (AUB)']$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - 5/6}{1 - 3/5}$$

$$= \frac{5}{12}$$

Example 15.19: The odds in favour of an event is 2 : 3 and the odds against another event is 3 : 7. Find the probability that only one of the two events occurs.

Solution: We denote the two events by A and B respectively. Then by (15.5) and (15.6), we have

$$P(A) = \frac{2}{2+3} = \frac{2}{5}$$

$$\text{and } P(B) = \frac{7}{7+3} = \frac{7}{10}$$

As A and B are independent, $P(A \cap B) = P(A) \times P(B)$

$$= \frac{2}{5} \times \frac{7}{10} = \frac{7}{25}$$

Probability that either only A occurs or only B occurs

$= P(A - B) + P(B - A)$

$= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$

$= P(A) + P(B) - 2\,P(A \cap B)$

$$= \frac{2}{5} + \frac{7}{10} - 2 \times \frac{7}{25}$$

$$= \frac{20 + 35 - 28}{50}$$

$$= \frac{27}{50}$$

Example 15.20: There are three boxes with the following compositions :

Box \ Colour	Blue	Red	White	Total
I	5	8	10	23
II	4	9	8	21
III	3	6	7	16

One ball is drawn from each box. What is the probability that they would be of the same colour?

Solution: Either the balls would be Blue or Red or White. Denoting Blue, Red and White balls by B, R and W respectively and the box by lower suffix, the required probability is

$$= P(B_1 \cap B_2 \cap B_3) + P(R_1 \cap R_2 \cap R_3) + P(W_1 \cap W_2 \cap W_3)$$

$$= P(B_1) \times P(B_2) \times P(B_3) + P(R_1) \times P(R_2) \times P(R_3) + P(W_1) \times P(W_2) \times P(W_3)$$

$$= \frac{5}{23} \times \frac{4}{21} \times \frac{3}{16} + \frac{8}{23} \times \frac{9}{21} \times \frac{6}{16} + \frac{10}{23} \times \frac{8}{21} \times \frac{7}{16}$$

$$= \frac{60+432+560}{7728}$$

$$= \frac{1052}{7728}$$

Example 15.21: Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates. What is the probability that Mr. Roy would be selected?

Solution: Denoting the three posts by A, B and C respectively, we have

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{5} \text{ and } P(C) = \frac{1}{10}$$

The probability that Mr. Roy would be selected (i.e. selected for at least one post).

$$= P(A \cup B \cup C)$$

$$= 1 - P[(A \cup B \cup C)']$$

$$= 1 - P(A' \cap B' \cap C') \quad \text{(by De-Morgan's Law)}$$

$$= 1 - P(A') \times P(B') \times P(C') \quad \text{(As A , B and C are independent, so are their complements)}$$

$$= 1 - \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{10}\right) = \frac{13}{25}$$

Example 15.22: The independent probabilities that the three sections of a costing department will encounter a computer error are 0.2, 0.3 and 0.1 per week respectively. What is the probability that there would be

(i) at least one computer error per week?

(ii) one and only one computer error per week?

Solution: Denoting the three sections by A, B and C respectively, the probabilities of encountering a computer error by these three sections are given by P(A) = 0.20, P(B) = 0.30 and P(C) = 0.10

(i) Probability that there would be at least one computer error per week.

$= 1 -$ Probability of having no computer error in any at the three sections.

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A') \times P(B') \times P(C')$ [Since A, B and C are independent]

$= 1 - (1 - 0.20) \times (1 - 0.30) \times (1 - 0.10)$

$= 0.50$

(ii) Probability of having one and only one computer error per week

$= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= P(A) \times P(B') \times P(C') + P(A') \times P(B) \times P(C') + P(A') \times P(B') \times P(C)$

$= 0.20 \times 0.70 \times 0.90 + 0.80 \times 0.30 \times 0.90 + 0.80 \times 0.70 \times 0.10$

$= 0.40$

Example 15.23: A lot of 10 electronic components is known to include 3 defective parts. If a sample of 4 components is selected at random from the lot, what is the probability that this sample does not contain more than one defectives?

Solution: Denoting defective component and non-defective components by D and D′ respectively, we have the following situation :

	D	D′	T
Lot	3	7	10
Sample (1)	0	4	4
(2)	1	3	4

Thus the required probability is given by

$$= ({}^{3}C_{0} \times {}^{7}C_{4} + {}^{3}C_{1} \times {}^{7}C_{3}) \,/\, {}^{10}C_{4}$$

$$= \frac{1 \times 35 + 3 \times 35}{210}$$

$$= \frac{2}{3}$$

Example 15.24: There are two urns containing 5 red and 6 white balls and 3 red and 7 white balls respectively. If two balls are drawn from the first urn without replacement and transferred to the second urn and then a draw of another two balls is made from it, what is the probability that both the balls drawn are red?

Solution: Since two balls are transferred from the first urn containing 5 red and 6 white balls to the second urn containing 3 red and 7 white balls, we are to consider the following cases :

Case A : Both the balls transferred are red. In this case, the second urn contains 5 red and 7 white balls.

Case B : The two balls transferred are of different colours. Then the second urn contains 4 red and 8 white balls.

Case C : Both the balls transferred are white. Now the second urn contains 3 red and 9 white balls.

The required probability is given by

$P(R \cap A) + P(R \cap B) + P(R \cap C)$

$= \quad P(R/A) \times P(A) + P(R/B) \times P(B) + P(R/C) \times P(C)$

$$= \frac{{}^5C_2}{{}^{12}C_2} \times \frac{{}^5C_2}{{}^{11}C_2} + \frac{{}^4C_2}{{}^{12}C_2} \times \frac{{}^5C_1 \times {}^6C_1}{{}^{11}C_2} + \frac{{}^3C_2}{{}^{12}C_2} \times \frac{{}^6C_2}{{}^{11}C_2}$$

$$= \frac{10}{66} \times \frac{10}{55} + \frac{6}{66} \times \frac{30}{55} + \frac{3}{66} \times \frac{15}{55}$$

$$= \frac{325}{66 \times 55} = \frac{65}{726}$$

Example 15.25: If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?

Solution: The first ball can be distributed to the 1st box or 2nd box or 3rd box i.e. it can be distributed in 3 ways. Similarly, the second ball also can be distributed in 3 ways. Thus the first two balls can be distributed in 3^2 ways. Proceeding in this way, we find that 8 balls can be distributed to 3 boxes in 3^8 ways which is the total number of elementary events.

Let A be the event that the first box contains 3 balls which implies that the remaining 5 balls must go to the remaining 2 boxes which, as we have already discussed, can be done in 2^5 ways. Since 3 balls out of 8 balls can be selected in ${}^8C_3$ ways, the event can occur in ${}^8C_3 \times 2^5$ ways, thus we have

$$P(A) = \frac{{}^8C_3 \times 2^5}{3^8}$$

$$= \frac{56 \times 32}{6561}$$

$$= \frac{1792}{6561}$$

Example 15.26: There are 3 boxes with the following composition :

Box I : 7 Red + 5 White + 4 Blue balls

Box II : 5 Red + 6 White + 3 Blue balls

Box III : 4 Red + 3 White + 2 Blue balls

One of the boxes is selected at random and a ball is drawn from it. What is the probability that the drawn ball is red?

Solution: Let A denote the event that the drawn ball is red. Since any of the 3 boxes may be drawn, we have $P(B_I) = P(B_{II}) = P(B_{III}) = \frac{1}{3}$

Also $P(R_1/B_{II})$ = probability of drawing a red ball from the first box

$$= \frac{7}{16}$$

$P(R_2 / B_{II}) = \frac{5}{14}$ and $P(R_3 / B_{III}) = \frac{4}{9}$

Thus we have

$$P(A) = P(R_1 \cap B_I) + P(R_2 \cap B_{II}) + P(R_3 \cap B_{III})$$

$$= P(R_1 / B_I) \times P(B_I) + P(R_2 / B_{II}) \times P(B_{II}) + P(R_3 / B_{III}) \times P(B_{III})$$

$$= \frac{7}{16} \times \frac{1}{3} + \frac{5}{14} \times \frac{1}{3} + \frac{4}{9} \times \frac{1}{3}$$

$$= \frac{7}{48} + \frac{5}{42} + \frac{4}{27}$$

$$= \frac{1249}{3024}$$

15.9 RANDOM VARIABLE - PROBABILITY DISTRIBUTION

A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment. A random variable is denoted by a capital letter. For example, if a coin is tossed three times and if X denotes the number of heads, then X is a random variable. In this case, the sample space is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

and we find that X = 0 if the sample point is TTT

X = 1 if the sample point is HTT, THT or TTH

X = 2 if the sample point is HHT, HTH or THH

and X = 3 if the sample point is HHH.

We can make a distinction between a discrete random variable and a continuous variable. A random variable defined on a discrete sample space is known as a discrete random variable and it can assume either only a finite number or a countably infinite number of values. The number of car accident, the number of heads etc. are examples of discrete random variables.

A continuous random variable, like height, weight etc. is a random variable defined on a continuous sample space and assuming an uncountably infinite number of values.

The probability distribution of a random variable may be defined as a statement expressing the different values taken by a random variable and the corresponding probabilities. Then if a random variable X assumes n finite values $X_1, X_2, X_3, \ldots\ldots, X_n$ with corresponding probabilities $P_1, P_2, P_3, \ldots\ldots, P_n$ such that

(i) $p_i \geq 0$ for every i …………… (15.33)

and (ii) $\sum p_i = 1$ (over all i) ………….. (15.34)

then the probability distribution of the random variable X is given by

Probability Distribution of X

X :	X_1	X_2	X_3	……X_n	Total
P :	P_1	P_2	P_3	……. P_n	1

For example, if an unbiased coin is tossed three times and if X denotes the number of heads then, as we have already discussed, X is a random variable and its probability distribution is given by

Probability Distribution of Head when a Coin is Tossed Thrice

X :	0	1	2	3	Total
P :	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

There are cases when it is possible to express the probability (P) as a function of X. In case X is a discrete variable and if such a function f(X) really exists, then f(X) is known as Probability Mass Function (PMF) of X, f(X), then, must satisfy the conditions:

(i) $f(X) \geq 0$ for every X …………… (15.35)

and (ii) $\sum_X f(X) = 1$ …………… (15.36)

Where f(X) is given by

$f(X) = P(X = x)$ …………… (15.37)

When x is a continuous random variable defined over an interval $[\alpha, \beta]$, where $\beta > \alpha$, then x can assume an infinite number of values from its interval and instead of assigning individual probability to every mass point x, we assign probabilities to interval of values. Such a function of x, provided it exists, is known as probability density function (pdf) of x. f(x) satisfies the following conditions:

(i) $f(x) \geq 0$ for $x \in [\alpha, \beta]$ .... (15.38)

(ii) $\int_{\alpha}^{\beta} f(x)dx = 1$ .... (15.39)

and the probability that x lies between two specified values a and b, where $\alpha \leq a < b \leq \beta$, is given by

$\int_{a}^{b} f(x)dx$ .... (15.40)

15.10 EXPECTED VALUE OF A RANDOM VARIABLE

Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities. Hence, if a random variable x assumes n values x_1, x_2, x_3 ...., x_n with corresponding probabilities p_1, p_2, p_3 ...., p_n, where p_i's satisy (15.33) and (15.34), then the expected value of x is given by

$\mu = E(x) = \sum p_i x_i$ .... (15.41)

Expected value of x^2 in given by

$E(x^2) = \sum p_i x_i^2$ .... (15.42)

In particular expected value of a monotonic function g (x) is given by

$E[g(x)] = \sum p_i g(x_i)$ .... (15.43)

Variance of x, to be denoted by , σ^2 is given by

$V(x) = \sigma^2 = E(x - \mu)^2$

$= E(x^2) - \mu^2$ .... (15.44)

The positive square root of variance is known as standard deviation and is denoted by σ.

If y = a + b x, for two random variables x and y and for a pair of constants a and b, then the mean i.e. expected value of y is given by

$\mu_y = a + b\ \mu_x$ .... (15.45)

and the standard deviation of y is

$\sigma_y = |b| \times \sigma_x$ .... (15.46)

When x is a discrete random variable with probability mass function f(x), then its expected value is given by

$$\mu = \sum_{x} x f(x) \quad \text{.........} (15.47)$$

and its variance is

$$\sigma^2 = E(x^2) - \mu^2$$

Where $E(x^2) = \sum_{x} x^2 f(x)$ ....(15.48)

For a continuous random variable x defined in [–∞, ∞], its expected value (i.e. mean) and variance are given by

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx \quad \text{.........} (15.49)$$

and $\sigma^2 = E(x^2) - \mu^2$

where $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$ .... (15.50)

Properties of Expected Values

1. Expectation of a constant k is k

   i.e. E(k) = k for any constant k. ....(15.51)

2. Expectation of sum of two random variables is the sum of their expectations.

   i.e. E(x + y) = E(x) + E(y) for any two random variables x and y. .... (15.52)

3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.

   i.e. E(k x) = k.E(x) for any constant k .... (15.53)

4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.

   i.e. $E(xy) = E(x) \times E(y)$ .... (15.54)

   Whenever x and y are independent.

Example 15.27: An unbiased coin is tossed three times. Find the expected value of the number of heads and also its standard deviation.

Solution: If x denotes the number of heads when an unbiased coin is tossed three times, then the probability distribution of x is given by

X :	0	1	2	3
P :	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The expected value of x is given by

$$\mu = E(x) = \sum p_i x_i$$

$$= \frac{1}{8}\times 0 + \frac{3}{8}\times 1 + \frac{3}{8}\times 2 + \frac{1}{8}\times 3$$

$$= \frac{0+3+6+3}{8} = 1.50$$

Also $\quad E(x^2) = \sum p_i x_i^2$

$$= \frac{1}{8}\times 0^2 + \frac{3}{8}\times 1^2 + \frac{3}{8}\times 2^2 + \frac{1}{8}\times 3^2$$

$$= \frac{0+3+12+9}{8} = 3$$

$$= \sigma^2 = E(x^2) - \mu^2$$

$$= 3 - (1.50)^2$$

$$= 0.75$$

$$\therefore SD = \sigma = 0.87$$

Example 15.28: A random variable has the following probability distribution:

X :	4	5	7	8	10
P :	0.15	0.20	0.40	0.15	0.10

Find E $[x - E(x)]^2$. Also obtain v(3x –4)

Solution: The expected value of x is given by

$$E(x) = \sum p_i x_i$$

$$= 0.15 \times 4 + 0.20 \times 5 + 0.40 \times 7 + 0.15 \times 8 + 0.10 \times 10$$

$$= 6.60$$

Also, $\quad E[x - E(x)]^2 = \sum \mu_i^2 P_i \quad$ where = $\mu_i = x_i - E(x)$

Let $y = 3x - 4 = (-4) + (3)x$. Then variance of y = var y = $b^2 \times \sigma_x^2 = 9 \times \mu_x^2$ (From 15.46)

Table 15.1

Computation of E [x – E(x)]²

x_i	p_i	$\mu_i = x_i - E(x)$	μ_i^2	$\mu_i^2 p_i$
4	0.15	–2.60	6.76	1.014
5	0.20	–1.60	2.56	0.512
7	0.40	0.40	0.16	0.064
8	0.15	1.40	1.96	0.294
10	0.10	3.40	11.56	1.156
Total	1.00	–	–	3.040

Thus $E[x - E(x)]^2 = 3.04$

As $\mu_x^2 = 3.04$, $v(y) = 9 \times 3.04 = 27.36$

Example 15.29: In a business venture, a man can make a profit of ₹ 50,000 or incur a loss of ₹ 20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?

Solution: If the profit is denoted by x, then we have the following probability distribution of x:

X :	₹ 50,000	₹ –20,000
P :	0.75	0.25

Thus his expected profit

$E(x) = p_1x_1 + p_2x_2$

$= 0.75 \times$ ₹ 50,000 + 0.25 $\times$ (₹ – 20,000)

= ₹ 32,500

Example 15.30: A box contains 12 electric lamps of which 5 are defectives. A man selects three lamps at random. What is the expected number of defective lamps in his selection?

Solution: Let x denote the number of defective lamps x can assume the values 0, 1, 2 and 3.

P(x = 0) = Prob. of having 0 defective out of 5 defectives and 3 non defective out of 7 non defectives

$$= \frac{{}^5C_0 \times {}^7C_3}{{}^{12}C_3} = \frac{35}{220}$$

Similarly $$P(x = 1) = \frac{{}^5C_1 \times {}^7C_2}{{}^{12}C_3} = \frac{105}{220}$$

$$P(x = 2) = \frac{{}^5C_2 \times {}^7C_1}{{}^{12}C_3} = \frac{70}{220}$$

and $$P(x = 3) = \frac{{}^5C_3 \times {}^7C_0}{{}^{12}C_3} = \frac{10}{220}$$

Probability Distribution of No. of Defective Lamp

X :	0	1	2	3
P :	$\frac{35}{220}$	$\frac{105}{220}$	$\frac{70}{220}$	$\frac{10}{220}$

Thus the expected number of defectives is given by

$$\frac{35}{220} \times 0 + \frac{105}{220} \times 1 + \frac{70}{220} \times 2 + \frac{10}{220} \times 3$$

= 1.25

Example 15.31: Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation? If he is asked to pay ₹ 400 for participating in the game, would he consider it a fair game and participate?

Solution: We denote the amount by x. Then x assumes the value 2 x ₹ 500 i.e. ₹ 1000 if 2 white balls are drawn, the value ₹ 500 + ₹ 200 i.e. ₹ 700 if 1 white and 1 red balls are drawn and the value 2 x ₹ 200 i.e. ₹ 400 if 2 red balls are drawn. The respective probabilities are given by

$$P(WW) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$$

$$P(WR) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28}$$

and $$P(RR) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$$

Probability Distribution of x

X :	₹ 1000	₹ 700	₹ 400
P :	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{10}{28}$

Hence E(x) $= \frac{3}{28} \times$ ₹ 1000 + $\frac{15}{28} \times$ ₹ 700 + $\frac{10}{28} \times$ ₹ 400

$$= \frac{₹\,3000 + ₹\,10500 + ₹\,4000}{28}$$

$=$ ₹ 625 > 400. Therefore the game is fair and he would participate.

Example 15.32: A number is selected at random from a set containing the first 100 natural numbers and another number is selected at random from another set containing the first 200 natural numbers. What is the expected value of the product?

Solution: We denote the number selected from the first set by x and the number selected from the second set by y. Since the selections are independent of each other, the expected value of the product is given by

$$E(xy) = E(x) \times E(y) \qquad \text{.......... (1)}$$

Now x can assume any value between 1 to 100 with the same probability 1/100 and as such the probability distribution of x is given by

X :	1	2	3	100
P :	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	.... $\frac{1}{100}$

$$\text{Thus } E(x) = \frac{1}{100} \times 1 + \frac{1}{100} \times 2 + \frac{1}{100} \times 3 + \ldots\ldots\ldots\ldots \frac{1}{100} \times 100$$

$$= \frac{1+2+3+\ldots\ldots\ldots+100}{100}$$

$$= \frac{100 \times 101}{2 \times 100} \qquad \left[\text{Since } 1+2+\ldots\ldots + n = \frac{n(n+1)}{2}\right]$$

$$= \frac{101}{2}$$

Similarly, $E(y) = \frac{201}{2}$

$$\therefore E(xy) = \frac{101}{2} \times \frac{201}{2} \qquad \text{[From (1)]}$$

$$= \frac{20301}{4}$$

$$= 5075.25$$

Example 15.33: A dice is thrown repeatedly till a 'six' appears. Write down the sample space. Also find the expected number of throws.

Solution: Let p denote the probability of getting a six and q = 1 – p, the probability of not getting a six. If the dice is unbiased then

$p = \frac{1}{6}$ and $q = \frac{5}{6}$

If a six obtained with the very first throw then the experiment ends and the probability of getting a six, as we have already seen, is p. However, if the first throw does not produce a six, the dice is thrown again and if a six appears with the second throw, the experiment ends. The probability of getting a six preceded by a non–six is qp. If the second thrown does not yield a six, we go for a third throw and if the third throw produces a six, the experiment ends and the probability of getting a Six in the third attempt is q^2p. The experiment is carried on and we get the following countably infinite sample space.

$S = \{ p, qp, q^2p, q^3p, \ldots\}$

If x denotes the number of throws necessary to produce a six, then x is a random variable with the following probability distribution :

X :	1	2	3	4 ....
P :	p	qp	q^2p	q^3p ....

Thus $E(x) = p \times 1 + qp \times 2 + q^2p \times 3 + q^3p \times 4 + \ldots$

$= p(1+ 2q + 3q^2 + 4q^3 + \ldots)$

$= p\,(1 - q)^{-2}$

$= \frac{p}{p^2}$ (as $1-q = p$)

$= \frac{1}{p}$

In case of an unbiased dice, $p = 1/6$ and $E(x) = 6$

Example 15.34: A random variable x has the following probability distribution :

X	:	0	1	2	3	4	5	6	7
P(X)	:	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2+k$

Find (i) the value of k

(ii) $P(x < 3)$

(iii) $P(x \geq 4)$

(iv) $P(2 < x \leq 5)$

Solution: By virtue of (15.36), we have

$\sum P(x) = 1$

$\Rightarrow \quad 0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1$

$\Rightarrow$ $10k^2 + 9k - 1 = 0$

$\Rightarrow$ $(k + 1)(10k - 1) = 0$

$\Rightarrow$ $k = 1/10$ (as $k \neq -1$ by virtue of (15.36))

(i) Thus the value of k is 0.10

(ii) $P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$

$= 0 + 2k + 3k$

$= 5k$

$= 0.50$ (as $k = 0.10$)

(iii) $P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7)$

$= 2k + k^2 + 7k^2 + (2k^2 + k)$

$= 10k^2 + 3k$

$= 10 \text{ x } (0.10)^2 + 3 \times 0.10$

$= 0.40$

(iv) $P(2 < x \leq 5) = P(x = 3) + P(x = 4) + P(x = 5)$

$= k + 2k + k^2$

$= k^2 + 3k$

$= (0.10)^2 + 3 \times 0.10$

$= 0.31$

SUMMARY

- **Experiment:** An experiment may be described as a performance that produces certain results.
- **Random Experiment:** An experiment is defined to be random if the results of the experiment depend on chance only.
- **Events:** The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types:

  (i) Simple or Elementary,

  (ii) Composite or Compound.
- **Mutually Exclusive Events or Incompatible Events:** A set of events $A_1, A_2, A_3, \ldots\ldots$ is known to be mutually exclusive if not more than one of them can occur simultaneously
- **Exhaustive Events:** The events $A_1, A_2, A_3, \ldots\ldots\ldots\ldots$ are known to form an exhaustive set if one of these events must necessarily occur.

- **Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:** The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events.
- The probability of occurrence of the event A is defined as the ratio of the number of events favourable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{n_A}{n} = \frac{\textit{No.}\text{ of equally likely events favourable to A}}{\text{Total no. of equally likely events}}$$

(a) The probability of an event lies between 0 and 1, both inclusive.

i.e. $0 \le P(A) \le 1$

When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.

(b) Non-occurrence of event A is denoted by A′ or A^C or $\bar{A}$ and it is known as complimentary event of A. The event A along with its complimentary A′ forms a set of mutually exclusive and exhaustive events.

i.e. $P(A) + P(A') = 1$

$\Rightarrow \quad P(A') = 1 - P(A)$

$$1 - \frac{m_A}{m}$$

$$= \frac{m - m_A}{m}$$

(c) The ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event A and its inverse ratio is known as odds against the event A.

i.e. odds in favour of A $= m_A : (m - m_A)$

and odds against A $= (m - m_A) : m_A$

(d) For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.

i.e. $P(A \cup B)$

or $P(A + B) = P(A) + P(B)$

(e) For any $K(\ge 2)$ mutually exclusive events $A_1, A_2, A_3 \ldots, A_K$ the probability that at least one of them occurs is given by the sum of the individual probabilities of the K events.

i.e. $P(A_1 \cup A_2 \cup \ldots \cup A_K) = P(A_1) + P(A_2) + \ldots . P(A_K)$

(f) For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(g) For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

(h) For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred

i.e. $P(A \cap B) = P(A) \times P(B/A)$ Provided $P(A) > 0$

(i) Compound Probability or Joint Probability

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

(j) For any three events A, B and C, the probability that they occur jointly is given by $P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B))$ Provided $P(A \cap B) > 0$

(k) $$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

(l) $$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

(m) $$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{P(A \cup B)'}{P(B')}$$ [by De-Morgan's Law $A' \cap B' = (AUB)'$]

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

- A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.
- Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

When x is a discrete random variable with probability mass function f(x), then its expected value is given by

$$\mu = \sum_{x} x f(x)$$

and its variance is

$$\sigma^2 = E(x^2) - \mu^2$$

Where $E(x^2) = \sum_{x} x^2 f(x)$

For a continuous random variable x defined in [–∞, ∞], its expected value (i.e. mean) and variance are given by

$$E(x) = \int_{-\infty}^{\infty} x f(x)dx$$

$$\text{and } \sigma^2 = E(x^2) - \mu^2$$

$$\text{where } E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

Properties of Expected Values

(i) Expectation of a constant k is k

i.e. E(k) = k for any constant k.

(ii) Expectation of sum of two random variables is the sum of their expectations.

i.e. E(x + y) = E(x) + E(y) for any two random variables x and y.

(iii) Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.

i.e. E(k x) = k.E(x) for any constant k

(iv) Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.

i.e. $E(xy) = E(x) \times E(y)$

Whenever x and y are independent.

EXERCISE

Set A

Write down the correct answers. Each question carRies 1 mark.

1. Initially, probability was a branch of
   (a) Physics (b) Statistics
   (c) Mathematics (d) Economics.
2. Two broad divisions of probability are
   (a) Subjective probability and objective probability
   (b) Deductive probability and non–deductive probability
   (c) Statistical probability and Mathematical probability
   (d) None of these.
3. Subjective probability may be used in
   (a) Mathematics (b) Statistics
   (c) Management (d) Accountancy.
4. An experiment is known to be random if the results of the experiment
   (a) Can not be predicted (b) Can be predicted
   (c) Can be split into further experiments (d) Can be selected at random.
5. An event that can be split into further events is known as
   (a) Complex event (b) Mixed event
   (c) Simple event (d) Composite event.
6. Which of the following pairs of events are mutually exclusive?
   (a) A : The student reads in a school. B : He studies Philosophy.
   (b) A : Raju was born in India. B : He is a fine Engineer.
   (c) A : Ruma is 16 years old. B : She is a good singer.
   (d) A : Peter is under 15 years of age. B : Peter is a voter of Kolkata.
7. If P(A) = P(B), then
   (a) A and B are the same events (b) A and B must be same events
   (c) A and B may be different events (d) A and B are mutually exclusive events.
8. If $P(A \cap B) = 0$, then the two events A and B are
   (a) Mutually exclusive (b) Exhaustive
   (c) Equally likely (d) Independent.

9. If for two events A and B, P(AUB) = 1, then A and B are
   (a) Mutually exclusive events (b) Equally likely events
   (c) Exhaustive events (d) Dependent events.
10. If an unbiased coin is tossed once, then the two events Head and Tail are
   (a) Mutually exclusive (b) Exhaustive
   (c) Equally likely (d) All these (a), (b) and (c).
11. If P(A) = P(B), then the two events A and B are
   (a) Independent (b) Dependent
   (c) Equally likely (d) Both (a) and (c).
12. If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are
   (a) Independent (b) Dependent
   (c) Not equally likely (d) Not exhaustive.
13. If P(A/B) = P(A), then
   (a) A is independent of B (b) B is independent of A
   (c) B is dependent of A (d) Both (a) and (b).
14. If two events A and B are independent, then
   (a) A and the complement of B are independent
   (b) B and the complement of A are independent
   (c) Complements of A and B are independent
   (d) All of these (a), (b) and (c).
15. If two events A and B are independent, then
   (a) They can be mutually exclusive (b) They can not be mutually exclusive
   (c) They can not be exhaustive (d) Both (b) and (c).
16. If two events A and B are mutually exclusive, then
   (a) They are always independent (b) They may be independent
   (c) They can not be independent (d) They can not be equally likely.
17. If a coin is tossed twice, then the events 'occurrence of one head', 'occurrence of 2 heads' and 'occurrence of no head' are
   (a) Independent (b) Equally likely
   (c) Not equally likely (d) Both (a) and (b).
18. The probability of an event can assume any value between
   (a) – 1 and 1 (b) 0 and 1, including 0 and 1
   (c) – 1 and 0 (d) none of these.

19. If P(A) = 0, then the event A

(a) will never happen (b) will always happen

(c) may happen (d) may not happen.

20. If P(A) = 1, then the event A is known as

(a) symmetric event (b) dependent event

(c) improbable event (d) sure event.

21. If p : q are the odds in favour of an event, then the probability of that event is

(a) $\frac{p}{q}$ (b) $\frac{p}{p+q}$

(c) $\frac{q}{p+q}$ (d) none of these.

22. If P(A) = 5/9, then the odds against the event A is

(a) 5 : 9 (b) 5 : 4

(c) 4 : 5 (d) 5 : 14

23. If A, B and C are mutually exclusive and exhaustive events, then P(A) + P(B) + P(C) equals to

(a) $\frac{1}{3}$ (b) 1

(c) 0 (d) any value between 0 and 1.

24. If A denotes that a student reads in a school and B denotes that he plays cricket, then

(a) $P(A \cap B) = 1$ (b) $P(A \cup B) = 1$

(c) $P(A \cap B) = 0$ (d) P(A) = P(B).

25. P(B/A) is defined only when

(a) A is a sure event (b) B is a sure event

(c) A is not an impossible event (d) B is an impossible event.

26. P(A/B') is defined only when

(a) B is not a sure event (b) B is a sure event

(c) B is an impossible event (d) B is not an impossible event.

27. For two events A and B, $P(A \cup B) = P(A) + P(B)$ only when

(a) A and B are equally likely events (b) A and B are exhaustive events

(c) A and B are mutually independent (d) A and B are mutually exclusive.

28. Addition Theorem of Probability states that for any two events A and B,

(a) $P(A \cup B) = P(A) + P(B)$ (b) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (d) $P(A \cup B) = P(A) \times P(B)$

29. For any two events A and B,

(a) $P(A) + P(B) > P(A \cap B)$ (b) $P(A) + P(B) < P(A \cap B)$

(c) $P(A) + P(B) \geq P(A \cap B)$ (d) $P(A) \times P(B) \leq P(A \cap B)$

30. For any two events A and B,

(a) $P(A–B) = P(A) - P(B)$ (b) $P(A–B) = P(A) - P(A \cap B)$

(c) $P(A–B) = P(B) - P(A \cap B)$ (d) $P(B–A) = P(B) + P(A \cap B)$.

31. The limitations of the classical definition of probability

(a) it is applicable when the total number of elementary events is finite

(b) it is applicable if the elementary events are equally likely

(c) it is applicable if the elementary events are mutually independent

(d) (a) and (b).

32. According to the statistical definition of probability, the probability of an event A is the

(a) limiting value of the ratio of the no. of times the event A occurs to the number of times the experiment is repeated

(b) the ratio of the frequency of the occurrences of A to the total frequency

(c) the ratio of the frequency of the occurrences of A to the non-occurrence of A

(d) the ratio of the favourable elementary events to A to the total number of elementary events.

33. The Theorem of Compound Probability states that for any two events A and B.

(a) $P(A \cap B) = P(A) \times P(B/A)$ (b) $P(A \cup B) = P(A) \times P(B/A)$

(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(A \cup B) = P(B) + P(B) - P(A \cap B)$.

34. If A and B are mutually exclusive events, then

(a) $P(A) = P(A–B)$. (b) $P(B) = P(A–B)$.

(c) $P(A) = P(A \cap B)$. (d) $P(B) = P(A \cap B)$.

35. If $P(A–B) = P(B–A)$, then the two events A and B satisfy the condition

(a) $P(A) = P(B)$. (b) $P(A) + P(B) = 1$

(c) $P(A \cap B) = 0$ (d) $P(A \cup B) = 1$

36. The number of conditions to be satisfied by three events A, B and C for complete independence is

(a) 2 (b) 3

(c) 4 (d) any number.

37. If two events A and B are independent, then $P(A \cap B)$

(a) equals to $P(A) + P(B)$
(b) equals to $P(A) \times P(B)$
(c) equals to $P(A) \times P(B/A)$
(d) equals to $P(B) \times P(A/B)$.

38. Values of a random variable are

(a) always positive numbers.
(b) always positive real numbers.
(c) real numbers.
(d) natural numbers.

39. Expected value of a random variable

(a) is always positive
(b) may be positive or negative
(c) may be positive or negative or zero
(d) can never be zero.

40. If all the values taken by a random variable are equal then

(a) its expected value is zero
(b) its standard deviation is zero
(c) its standard deviation is positive
(d) its standard deviation is a real number.

41. If x and y are independent, then

(a) $E(xy) = E(x) \times E(y)$
(b) $E(xy) = E(x) + E(y)$
(c) $E(x - y) = E(x) + E(y)$
(d) $E(x - y) = E(x) + x\,E(y)$

42. If a random variable x assumes the values x_1, x_2, x_3, x_4 with corresponding probabilities p_1, p_2, p_3, p_4 then the expected value of x is

(a) $p_1 + p_2 + p_3 + p_4$
(b) $x_1 p_1 + x_2 p_3 + x_3 p_2 + x_4 p_4$
(c) $p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$
(d) none of these.

43. f(x), the probability mass function of a random variable x satisfies

(a) $f(x) > 0$
(b) $\sum_x f(x) = 1$
(c) both (a) and (b)
(d) $f(x) \geq 0$ and $\sum_x f(x) = 1$

44. Variance of a random variable x is given by

(a) $E(x - \mu)^2$
(b) $E[x - E(x)]^2$
(c) $E(x^2 - \mu)$
(d) (a) and (b)

45. If two random variables x and y are related by $y = 2 - 3x$, then the SD of y is given by

(a) $-3 \times$ SD of x
(b) $3 \times$ SD of x.
(c) $9 \times$ SD of x
(d) $2 \times$ SD of x.

46. Probability of getting a head when two unbiased coins are tossed simultaneously is

(a) 0.25
(b) 0.50
(c) 0.20
(d) 0.75

47. If an unbiased coin is tossed twice, the probability of obtaining at least one tail is

(a) 0.25
(b) 0.50
(c) 0.75
(d) 1.00

48. If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is

(a) 1:2 (b) 2:1

(c) 1:3 (d) 3:1

49. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is

(a) 0.30 (b) 0.70

(c) 0.25 (d) 0.20

50. A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively. What is P $(A \cap B \cap C)$?

(a) 0.400 (b) 0.240

(c) 0.024 (d) 0.500

51. If two letters are taken at random from the word HOME, what is the Probability that none of the letters would be vowels?

(a) 1/6 (b) 1/2

(c) 1/3 (d) 1/4

52. If a card is drawn at random from a pack of 52 cards, what is the chance of getting a Spade or an ace?

(a) 4/13 (b) 5/13

(c) 0.25 (d) 0.20

53. If x and y are random variables having expected values as 4.5 and 2.5 respectively, then the expected value of (x–y) is

(a) 2 (b) 7

(c) 6 (d) 0

54. If variance of a random variable x is 23, then what is the variance of 2x+10?

(a) 56 (b) 33

(c) 46 (d) 92

55. What is the probability of having at least one 'six' from 3 throws of a perfect die?

(a) 5/6 (b) $(5/6)^3$

(c) $1-(1/6)^3$ (d) $1-(5/6)^3$

Set B

Write down the correct answers. Each question carries 2 marks.

1. Two balls are drawn from a bag containing 5 white and 7 black balls at random. What is the probability that they would be of different colours?

(a) 35/66 (b) 30/66

(c) 12/66 (d) None of these

2. What is the chance of throwing at least 7 in a single cast with 2 dice?

(a) 5/12 (b) 7/12

(c) 1/4 (d) 17/36

3. What is the chance of getting at least one defective item if 3 items are drawn randomly from a lot containing 6 items of which 2 are defective item?

(a) 0.30 (b) 0.20

(c) 0.80 (d) 0.50

4. If two unbiased dice are rolled together, what is the probability of getting no difference of points?

(a) 1/2 (b) 1/3

(c) 1/5 (d) 1/6

5. If A, B and C are mutually exclusive independent and exhaustive events then what is the probability that they occur simultaneously?

(a) 1 (b) 0.50

(c) 0 (d) any value between 0 and 1

6. There are 10 balls numbered from 1 to 10 in a box. If one of them is selected at random, what is the probability that the number printed on the ball would be an odd number greater that 4?

(a) 0.50 (b) 0.40

(c) 0.60 (d) 0.30

7. Following are the wages of 8 workers in rupees:

50, 62, 40, 70, 45, 56, 32, 45

If one of the workers is selected at random, what is the probability that his wage would be lower than the average wage?

(a) 0.625 (b) 0.500

(c) 0.375 (d) 0.450

8. A, B and C are three mutually exclusive and exhaustive events such that P (A) = 2 P (B) = 3P(C). What is P (B)?

(a) 6/11 (b) 3/11

(c) 1/6 (d) 1/3

9. For two events A and B, P (B) = 0.3, P (A but not B) = 0.4 and P (not A) = 0.6. The events A and B are

(a) exhaustive (b) independent

(c) equally likely (d) mutually exclusive

10. A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6 ?

(a) 0.30 (b) 0.25

(c) 0.20 (d) 1/3

11. Given that for two events A and B, P (A) = 3/5, P (B) = 2/3 and P ($A \cup B$) = 3/4, what is P (A/B)?

(a) 0.655 (b) 13/60

(c) 31/60 (d) 0.775

12. For two independent events A and B, what is P (A+B), given P(A) = 3/5 and P(B) = 2/3?

(a) 11/15 (b) 13/15

(c) 7/15 (d) 0.65

13. If P (A) = p and P (B) = q, then

(a) $P(A/B) \le p/q$ (b) $P(A/B) < p/q$

(c) $P(A/B) \le q/p$ (d) None of these

14. If P ($\bar{A} \cup \bar{B}$) = 5/6, P(A) = ½ and P ($\bar{B}$) = 2/3, , what is P ($A \cup B$) ?

(a) 1/3 (b) 5/6

(c) 2/3 (d) 4/9

15. If for two independent events A and B, P ($A \cup B$) = 2/3 and P (A) = 2/5, what is P (B)?

(a) 4/15 (b) 4/9

(c) 5/9 (d) 7/15

16. If P (A) = 2/3, P (B) =3/4, P (A/B) = 2/3, then what is P (B / A)?

(a) 1/3 (b) 2/3

(c) 3/4 (d) 1/2

17. If P (A) = a, P (B) = b and P ($A \cap B$) = c then the expression of P ($A' \cap B'$) in terms of a, b and c is

(a) 1 – a – b – c (b) a + b – c

(c) 1 + a – b – c (d) 1 – a – b + c

18. For three events A, B and C, the probability that only A occur is

(a) P (A) (b) $P(A \cup B \cup C)$

(c) $P(A' \cap B \cap C)$ (d) $P(A \cap B' \cap C')$

19. It is given that a family of 2 children has a girl, what is the probability that the other child is also a girl ?

(a) 0.50 (b) 0.75

(c) 1/3 (d) 2/3

20. Two coins are tossed simultaneously. What is the probability that the second coin would show a tail given that the first coin has shown a head?

(a) 0.50 (b) 0.25

(c) 0.75 (d) 0.125

21. If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is

(a) 1.50 (b) 3

(c) 0.90 (d) 1

22. If two random variables x and y are related as $y = -3x + 4$ and standard deviation of x is 2, then the standard deviation of y is

(a) – 6 (b) 6

(c) 18 (d) 3.50

23. If $2x + 3y + 4 = 0$ and $v(x) = 6$ then v (y) is

(a) 8/3 (b) 9

(c) – 9 (d) 6

Set C

Write down the correct answers. Each question carries 5 marks.

1. What is the probability that a leap year selected at random would contain 53 Saturdays?

(a) 1/7 (b) 2/7

(c) 1/12 (d) 1/4

2. If an unbiased coin is tossed three times, what is the probability of getting more that one head?

(a) 1/8 (b) 3/8

(c) 1/2 (d) 1/3

3. If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

(a) 0.25 (b) 0.50

(c) 0.75 (d) 0.80

4. What is the probability that 4 children selected at random would have different

birthdays?

(a) $\frac{364 \times 363 \times 362}{(365)^3}$ (b) $\frac{6 \times 5 \times 4}{7^3}$

(c) 1/365 (d) $(1/7)^3$

5. A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made (i) with replacement (ii) without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively

(a) 6/321 and 3/926 (b) 1/20 and 1/30

(c) 35/144 and 35/108 (d) 7/968 and 5/264

6. There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of same colour?

(a) 89/729 (b) 97/729

(c) 82/729 (d) 23/32

7. A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?

(a) 0.25 (b) 0.32

(c) 0.22 (d) 0.33

8. A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is

(a) 5/223 (b) 6/257

(c) 7/429 (d) 3/548

9. There are two boxes containing 5 white and 6 blue balls and 3 white and 7 blue balls respectively. If one of the the boxes is selected at random and a ball is drawn from it, then the probability that the ball is blue is

(a) 115/227 (b) 83/250

(c) 137/220 (d) 127/250

10. A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

(a) 4/15 (b) 7/8

(c) 8/15 (d) 11/15

11. There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

(a) 0.425 (b) 0.456

(c) 0.392 (d) 0.388

12. Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

(a) 0.325 (b) 0.400

(c) 0.925 (d) 0.075

13. There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the second urn. The probability that the second ball would be red is

(a) 7/20 (b) 35/88

(c) 17/52 (d) 3/20

14. For a group of students, 30 %, 40% and 50% failed in Physics , Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/6

15. A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

(a) 1/3 (b) 2/3

(c) 13/15 (d) 3/15

16. 8 identical balls are placed at random in three bags. What is the probability that the first bag will contain 3 balls?

(a) 0.2731 (b) 0.3256

(c) 0.1924 (d) 0.3443

17. X and Y stand in a line with 6 other people. What is the probability that there are 3 persons between them?

(a) 1/5 (b) 1/6

(c) 1/7 (d) 1/3

18. Given that $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$, what is $P(A'/B')$

(a) 1/2 (b) 7/8

(c) 5/8 (d) 2/3

19. Four digits 1, 2, 4 and 6 are selected at random to form a four digit number. What is the probability that the number so formed, would be divisible by 4?

(a) 1/2 (b) 1/5

(c) 1/4 (d) 1/3

20. The probability distribution of a random variable x is given below:

x :	1	2	4	5	6
P :	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x?

(a) 1.49 (b) 1.56

(c) 1.69 (d) 1.72

21. A packet of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

(a) 1.20 (b) 1.21

(c) 1.69 (d) 1.72

22. The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statements

(a) 170 (b) 176

(c) 178 (d) 180

23. A bag contains 6 white and 4 red balls. If a person draws 2 balls and receives ₹ 10 and ₹ 20 for a white and red balls respectively, then his expected amount is

(a) ₹ 25 (b) ₹ 26

(c) ₹ 29 (d) ₹ 28

24. The probability distribution of a random variable is as follows:

x :	1	2	4	6	8
P :	k	2k	3k	3k	k

The variance of x is

(a) 2.1 (b) 4.41

(c) 2.32 (d) 2.47

ANSWERS

Set A

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(d)	**5.**	(d)	**6.**	(d)
7.	(c)	**8.**	(a)	**9.**	(c)	**10.**	(d)	**11.**	(c)	**12.**	(b)
13.	(d)	**14.**	(d)	**15.**	(b)	**16.**	(c)	**17.**	(c)	**18.**	(b)
19.	(a)	**20.**	(d)	**21.**	(b)	**22.**	(c)	**23.**	(b)	**24.**	(c)
25.	(c)	**26.**	(a)	**27.**	(d)	**28.**	(c)	**29.**	(c)	**30.**	(b)
31.	(d)	**32.**	(a)	**33.**	(a)	**34.**	(a)	**35.**	(a)	**36.**	(c)
37.	(b)	**38.**	(c)	**39.**	(c)	**40**	(b)	**41.**	(a)	**42.**	(c)
43.	(d)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(c)	**48.**	(a)
49.	(b)	**50.**	(c)	**51.**	(a)	**52.**	(a)	**53.**	(a)	**54.**	(d)
55.	(d)										

Set B

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(d)
7.	(b)	**8.**	(b)	**9.**	(d)	**10.**	(d)	**11.**	(d)	**12.**	(b)
13.	(a)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(d)	**18.**	(d)
19.	(c)	**20.**	(a)	**21.**	(c)	**22.**	(b)	**23.**	(a)		

Set C

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(a)
7.	(c)	**8.**	(c)	**9.**	(c)	**10.**	(d)	**11.**	(d)	**12.**	(b)
13.	(b)	**14.**	(a)	**15.**	(c)	**16.**	(a)	**17.**	(c)	**18.**	(c)
19.	(d)	**20.**	(c)	**21.**	(a)	**22.**	(c)	**23.**	(d)	**24.**	(b)

ADDITIONAL QUESTION BANK

1. All possible outcomes of a random experiment forms the
   (a) events (b) sample space (c) both (d) none
2. If one of outcomes cannot be expected to occur in preference to the other in an experiment the events are
   (a) simple events (b) compound events
   (c) favourable events (d) equally likely events
3. If two events cannot occur simultaneously in the same trial then they are
   (a) mutually exclusive events (b) simple events
   (c) favourable events (d) none
4. When the number of cases favourable to the event A is none then P(A) is equal to
   (a) 1 (b) 0 (c) $\frac{1}{2}$ (d) none
5. A card is drawn from a well-shuffled pack of playing cards. The probability that it is a spade is
   (a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) $\frac{3}{13}$ (d) none
6. A card is drawn from a well-shuffled pack of playing cards. The probability that it is a king is
   (a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) $\frac{4}{13}$ (d) none
7. A card is drawn from a well-shuffled pack of playing cards. The probability that it is the ace of clubs is
   (a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) $\frac{1}{52}$ (d) none
8. In a single throw with two dice the probability of getting a sum of five on the two dice is
   (a) $\frac{1}{9}$ (b) $\frac{5}{36}$ (c) $\frac{5}{9}$ (d) none
9. In a single throw with two dice, the probability of getting a sum of six on the two dice is
   (a) $\frac{1}{9}$ (b) $\frac{5}{36}$ (c) $\frac{5}{9}$ (d) none
10. The probability that exactly one head appears in a single throw of two fair coins is
    (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) none
11. The probability that at least one head appears in a single throw of three fair coins is
    (a) $\frac{1}{8}$ (b) $\frac{7}{8}$ (c) $\frac{1}{3}$ (d) none
12. The definition of probability fails when the no of possible outcomes of the experiment is infinite
    (a) True (b) false (c) both (d) none

13. The following table gives distribution of wages of 100 workers –

Wages (in ₹)	120–140	140–160	160–180	180–200	200–220	220–240	240–260
No. of workers	9	20	0	10	8	35	18

The probability that his wages are under ₹ 140 is
(a) 20/100 (b) 9/100 (c) 29/100 (d) none

14. An individual is selected at random from the above group. The probability that his wages are under ₹160 is
(a) 9/100 (b) 20/100 (c) 29/100 (d) none

15. For the above table the probability that his wages are above ₹ 200 is
(a) 43/100 (b) 35/100 (c) 53/100 (d) 61/100

16. For the above table the probability that his wages between ₹ 160 and 220 is
(a) 30/100 (b) 10/100 (c) 38/100 (d) 18/100

17. The table below shows the history of 1000 men :

Life (in years) :	60	70	80	90
No. survived :	1000	500	100	60

The probability that a man will survived to age 90 is
(a) 60/1000 (b) 160/1000 (c) 660/1000 (d) none

18. The terms "chance" and probability are synonymous
(a) True (b) false (c) both (d) none

19. If probability of drawing a spade from a well-shuffled pack of playing cards is $\frac{1}{4}$ then the probability that of the card drawn from a well-shuffled pack of playing cards is 'not a spade' is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

20. Sum of probability of events in sample space is
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) none

21. Sum of all probabilities of mutually exclusive and exhaustive events is equal to
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

22. Let a sample space be $S = \{X_1, X_2, X_3\}$ which of the following defines probability space on S ?
(a) $P(X_1)= \frac{1}{4}$, $P(X_2)= \frac{1}{3}$, $P(X_3)= \frac{1}{3}$ (b) $P(X_1)= 0$, $P(X_2)= \frac{1}{3}$, $P(X_3)= \frac{2}{3}$
(c) $P(X_1)= \frac{2}{3}$, $P(X_2)= \frac{1}{3}$, $P(X_3)= \frac{2}{3}$ (d) none

23. Let P be a probability function on $S = \{X_1, X_2, X_3\}$ if $P(X_1) = \frac{1}{4}$ and $P(X_3) = \frac{1}{3}$ then P (X_2) is equal to
(a) 5/12 (b) 7/12 (c) 3/4 (d) none
24. The chance of getting a sum of 10 in a single throw with two dice is
(a) 10/36 (b) 1/12 (c) 5/36 (d) none
25. The chance of getting a sum of 6 in a single throw with two dice is
(a) 3/36 (b) 4/36 (c) 6/36 (d) 5/36
26. P (B/A) defines the probability that event B occurs on the assumption that A has happened
(a) Yes (b) no (c) both (d) none
27. The complete group of all possible outcomes of a random experiment given an ________ set of events.
(a) mutually exclusive (b) exhaustive (c) both (d) none
28. When the event is 'certain' the probability of it is
(a) 0 (b) 1/2 (c) 1 (d) none
29. The classical definition of probability is based on the feasibility at subdividing the possible outcomes of the experiments into
(a) mutually exclusive and exhaustive
(b) mutually exclusive and equally likely
(c) exhaustive and equally likely
(d) mutually exclusive,exhaustive and equally likely cases.
30. Two unbiased coins are tossed. The probability of obtaining 'both heads' is
(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{3}{4}$ (d) none
31. Two unbiased coins are tossed. The probability of obtaining one head and one tail is
(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{3}{4}$ (d) none
32. Two unbiased coins are tossed. The probability of obtaining both tail is
(a) $\frac{2}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) none
33. Two unbiased coins are tossed. The probability of obtaining at least one head is
(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{3}{4}$ (d) none
34. When two unbiased coins are tossed, the probability of obtaining 3 heads is
(a) $\frac{2}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 0
35. When two unbiased coins are tossed, the probability of obtaining not more than 3 heads is
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

36. When two unbiased coins are tossed, the probability of getting both heads or both tails is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) none

37. Two dice with face marked 1, 2, 3, 4, 5, 6 are thrown simultaneously and the points on the dice are multiplied together. The probability that product is 12 is

(a) 4/36 (b) 5/36 (c) 12/36 (d) none

38. A bag contain 6 white and 5 black balls. One ball is drawn. The probability that it is white is

(a) 5/11 (b) 1 (c) 6/11 (d) 1/11

39. Probability of occurrence of at least one of the events A and B is denoted by

(a) P(AB) (b) P(A+B) (c) P(A/B) (d) none

40. Probability of occurrence of A as well as B is denoted by

(a) P(AB) (b) P(A+B) (c) P(A/B) (d) none

41. Which of the following relation is true ?

(a) $P(A)- P(A^C)= 1$ (b) $P(A)+ P(A^C)= 1$ (c) $P(A)\ P(A^C)= 1$ (d) none

42. If events A and B are mutually exclusive, the probability that either A or B occurs is given by

a) P (A+B)= P(A)– P(B) (b) P (A+B)=P(A)+ P(B)– P(AB)

c) P (A+B)= P(A)– P(B)+ P(AB) (d) P (A+B)= P(A)+ P(B)

43. The probability of occurrence of at least one of the 2 events A and B (which may not be mutually exclusive) is given by

a) P(A+B)= P(A)– P(B) (b) P(A+B)= P(A)+ P(B)– P(AB)

c) P(A+B)= P(A)– P(B)+ P(AB) (d) P(A+B)= P(A)+P (B)

44. If events A and B are independent, the probability of occurrence of A as well as B is given by

(a) P(AB)= P(A/B) (b) P(AB)= P(A)/P(B)

(c) P(AB)= P(A)P(B) (d) None

45. For the condition P(AB)= P(A)P(B)two events A and B are said to be

(a) dependent (b) independent (c) equally like (d) none

46. The conditional probability of an event B on the assumption that another event A has actually occurred is given by

(a) P(B/A)= P(AB)/P(A) (b) P(A/B)= P(AB)/ P(B)

(c) P(B/A)= P(AB) (d) P(A/B)= P(AB)/ P(A)P(B)

47. Given $P(A)= \frac{1}{2}$, $P(B)= \frac{1}{3}$, $P(AB)= \frac{1}{4}$, the value of P(A+B) is

a) $\frac{3}{4}$ b) $\frac{7}{12}$ c) $\frac{5}{6}$ d) $\frac{1}{6}$

48. Given P(A)= $\frac{1}{2}$, P(B)= $\frac{1}{3}$, P (AB)= $\frac{1}{4}$, the value of P (A/B) is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

49. If P (A)= $\frac{1}{3}$, P(B)= $\frac{1}{4}$, the events A & B are

a) not equally likely b) mutually exclusive
c) equally likely d) none

50. If events A and B are independent then
a) A^C and B^C are dependent b) A^C and B are dependent
c) A and B^C are dependent d) A^C and B^C are also independent

51. A card is drawn from each of two well-shuffled packs of cards.The probability that at least one of them is an ace is

a) $\frac{1}{69}$ b) $\frac{25}{169}$ c) $\frac{2}{13}$ d) none

52. When a die is tossed, the sample space is
a) S ={1,2,3,4,5} b) S ={1,2,3,4} c) S ={1,2,3,4,5,6} d) none

53. If P (A)= $\frac{1}{4}$, P(B)= $\frac{2}{5}$, P (A+B)= $\frac{1}{2}$ then P(AB)is equal to

a) $\frac{3}{4}$ b) $\frac{2}{20}$ c) $\frac{13}{20}$ d) $\frac{3}{20}$

54. If events A and B are independent and P(A)= 2/3 , P(B)= 3/5 then P(A+B)is equal to

a) $\frac{13}{15}$ b) $\frac{6}{15}$ c) $\frac{1}{15}$ d) none

55. The expected number of head in 100 tosses of an unbiased coin is
a) 100 b) 50 c) 25 d) none

56. A and B are two events such that P(A)= $\frac{1}{3}$, P(B) = $\frac{1}{4}$, P(A+B)= $\frac{1}{2}$, than P(B/A) is equal to

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) none

57. Probability mass function is always
a) 0 b) greater than 0
c) greater than equal to 0 d) less than 0

58. The sum of probability mass function is equal to
a) –1 b) 0 c) 1 d) none

59. When X is a continuous function f(x)is called
a) probability mass function b) probability density function
c) both d) none

60. Which of the following set of function define a probability space on S = {a_1, a_2, a_3}

a) $P(a_1)= \frac{1}{3}$, $P(a_2) = \frac{1}{2}$, $P(a_3)= \frac{1}{4}$ b) $P(a_1)= \frac{1}{3}$, $P(a_2)= \frac{1}{6}$, $P(a_3)= \frac{1}{2}$

c) $P(a_1)= P(a_2)= \frac{2}{3}$, $P(a_3)= \frac{1}{4}$ d) None

61. If P $(a_1)= 0$, $P(a_2)= \frac{1}{3}$, P $(a_3) = \frac{2}{3}$ then S = {a_1, a_2, a_3} is a probability space

a) true b) false c) both d) none

62. If two events are independent then

a) P(B/A)= P(AB) P(A) b) P(B/A)= P(AB) P(B)

c) P(B/A)= P(B) d) P(B/A)= P(A)

63. When expected value is negative the result is

a) favourable b) unfavourable

c) both d) none to the above

64. The expected value of X, the sum of the scores, when two dice are rolled is

a) 9 b) 8 c) 6 d) 7

65. Let A and B be the events with P(A)= 1/3, P(B) = 1/4 and P(AB)= 1/12 then P(A/B) is equal to

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{2}{3}$

66. Let A and B be the events with P(A)= 2/3, P(B)= 1/4 and P(AB)= 1/12 then P(B/A) is equal to

a) $\frac{7}{8}$ b) $\frac{1}{3}$ c) $\frac{1}{8}$ d) none

67. The odds in favour of one student passing a test are 3:7.The odds against another student passing at are 3:5.The probability that both pass is

a) $\frac{7}{16}$ b) $\frac{21}{80}$ c) $\frac{9}{80}$ d) $\frac{3}{16}$

68. The odds in favour of one student passing a test are 3:7.The odds against another student passing at are 3:5. The probability that both fail is

a) $\frac{7}{16}$ b) $\frac{21}{80}$ c) $\frac{9}{80}$ d) $\frac{3}{16}$

69. In formula P(B/A), P(A) is

a) greater than zero b) less than zero

c) equal to zero d) greater than equal to zero

70. Two events A and B are mutually exclusive means they are

a) not disjoint b) disjoint c) equally likely d) none

71. A bag contains 10 white and 10 black balls A ball is drawn from it. The probability that it will be white is

(a) $\frac{1}{10}$ (b) 1 (c) $\frac{1}{2}$ (d) none

72. Two dice are thrown at a time. The probability that the numbers shown are equal is

(a) $\frac{2}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) none

73. Two dice are thrown at a time. The probability that 'the difference of numbers shown is 1' is

(a) $\frac{11}{18}$ (b) $\frac{5}{18}$ (c) $\frac{7}{18}$ (d) none

74. Two dice are thrown together. The probability that 'the event the difference of numbers shown is 2' is

(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

75. The probability space in tossing two coins is

(a) {(H,H),(H,T),(T,H)} (b) {(H,T),(T,H),(T,T)}
(c) {(H,H),(H,T),(T,H), (T,T)} (d) none

76. The probability of drawing a white ball from a bag containing 3 white and 8 blue balls is

(a) 3/5 (b) 3/11 (c) 8/11 (d) none

77. Two dice are thrown together. The probability of the event that the sum of numbers shown is greater than 5 is

(a) 13/18 (b) 15/18 (c) 1 (d) none

78. A traffic census show that out of 1000 vehicles passing a junction point on a highway 600 turned to the right. The probability of an automobile turning the right is

(a) 2/5 (b) 3/5 (c) 4/5 (d) none

79. Three coins are tossed together. The probability of getting three tails is

(a) 5/8 (b) 3/8 (c) 1/8 (d) none

80. Three coins are tossed together.The probability of getting exactly two heads is

(a) 5/8 (b) 3/8 (c) 1/8 (d) none

81. Three coins are tossed together. The probability of getting at least two heads is

(a) 1/2 (b) 3/8 (c) 1/8 (d) none

82. 4 coins are tossed. The probability that there are 2 heads is

(a) 1/2 (b) 3/8 (c) 1/8 (d) none

83. If 4 coins are tossed. The chance that there should be two tails is

(a) 1/2 (b) 3/8 (c) 1/8 (d) none

84. If A is an event and A^C its complementary event then

(a) $P(A)=P(A^C)-1$ (b) $P(A^C)=1-P(A)$ (c) $P(A)=1+P(A^C)$ (d) none

85. If P(A)= 3/8, P(B)= 1/3 and P(AB)= $\frac{1}{4}$ then $P(A^C)$ is equal to

(a) 5/8 (b) 3/8 (c) 1/8 (d) none

86. If P(A)= 3/8, P(B)= 1/3 then $P(\overline{B})$ is equal to

(a) 1 (b) 1/3 (c) 2/3 (d) none

87. If P(A)= 3/8, P(B)= 1/3 and P(AB)= $\frac{1}{4}$ then P(A + B)is

(a) 13/24 (b) 11/24 (c) 17/24 (d) none

88. If P(A)= 1/5, P(B)= 1/2 and A and B are mutually exclusive then P(AB) is

(a) 7/10 (b) 3/10 (c) 1/5 (d) none

89. The probability of throwing more than 4 in a single throw from an ordinary die is

(a) 2/3 (b) 1/3 (c) 1 (d) none

90. The probability that a card drawn at random from the pack of playing cards may be either a queen or an ace is

(a) 2/13 (b) 11/13 (c) 9/13 (d) none

91. The chance of getting 7 or 11 in a throw of 2 dice is

(a) 7/9 (b) 5/9 (c) 2/9 (d) none

92. If the probability of a horse A winning a race is 1/6 and the probability of a horse B winning the same race is 1/4 , what is the probability that one of the horses will win

(a) 5/12 (b) 7/12 (c) 1/12 (d) none

93. If the probability of a horse A winning a race is 1/6 and the probability of a horse B winning the same race is 1/4 , What is the probability that none of them will win

(a) 5/12 (b) 7/12 (c) 1/12 (d) none

94. If P (A)= 7/8 then($P(A^C)$ is equal to

(a) 1 (b) 0 (c) 7/8 (d) 1/8

95. The value of P(S) were S is the sample space is

(a) –1 (b) 0 (c) 1 (d) none

96. A man can kill a bird once in three shots.The probabilities that a bird is not killed is

(a) 1/3 (b) 2/3 (c) 1 (d) 0

97. If on an average 9 shops out of 10 return safely to a port, the probability of one ship returns safely is

(a) 1/10 (b) 8/10 (c) 9/10 (d) none

98. If on an average 9 shops out of 10 return safely to a port, the probability of one ship does not reach safely is

(a) 1/10 (b) 8/10 (c) 9/10 (d) none

99. The probability of winning of a person is 6/11 and at a result he gets ₹ 77/-. The expectation of this person is

(a) ₹ 35/- (b) ₹ 42/- (c) ₹ 58/- (d) none

100. A family has 2 children. The probability that both of them are boys if it is known that one of them is a boy

(a) 1 (b) 1/2 (c) 3/4 (d) none

101. The Probability of the occurrence of a number greater then 2 in a throw of a die if it is known that only even numbers can occur is

(a) 1/3 (b) 1/2 (c) 2/3 (d) none

102. A player has 7 cards in hand of which 5 are red and of these five 2 are kings. A card is drawn at random. The probability that it is a king, it being known that it is red is

(a) 2/5 (b) 3/5 (c) 4/5 (d) none

103. In a class 40 % students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random. The probability that he reads Mathematics if it is known that he reads Biology is

(a) 2/5 (b) 3/5 (c) 4/5 (d) none

104. In a class 40 % students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random.The probability that he reads Biology if he reads Mathematics

(a) 7/8 (b) 1/8 (c) 3/8 (d) none

105. Probability of throwing an odd no with an ordinary six faced die is

(a) 1/2 (b) 1 (c) –1/2 (d) 0

106. For a event A which is certain, P (A) is equal to

(a) 1 (b) 0 (c) –1 (d) none

107. When none of the outcomes is favourable to the event then the event is said to be

(a) certain (b) sample (c) impossible (d) none

ANSWERS

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(b)
6.	(a)	**7.**	(c)	**8.**	(a)	**9.**	(b)	**10.**	(b)
11.	(b)	**12.**	(a)	**13.**	(b)	**14.**	(c)	**15.**	(d)
16.	(d)	**17.**	(a)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(d)	**22.**	(b)	**23.**	(a)	**24.**	(b)	**25.**	(d)
26.	(a)	**27.**	(b)	**28.**	(c)	**29.**	(d)	**30.**	(a)
31.	(b)	**32.**	(c)	**33.**	(c)	**34.**	(d)	**35.**	(c)
36.	(a)	**37.**	(a)	**38.**	(c)	**39.**	(b)	**40.**	(a)
41.	(b)	**42.**	(d)	**43.**	(b)	**44.**	(c)	**45.**	(b)
46.	(a)	**47.**	(b)	**48.**	(d)	**49.**	(a)	**50.**	(d)
51.	(b)	**52.**	(c)	**53.**	(d)	**54.**	(a)	**55.**	(b)
56.	(a)	**57.**	(c)	**58.**	(c)	**59.**	(b)	**60.**	(b)
61.	(a)	**62.**	(c)	**63.**	(b)	**64.**	(d)	**65.**	(a)
66.	(c)	**67.**	(d)	**68.**	(b)	**69.**	(a)	**70.**	(b)
71.	(c)	**72.**	(c)	**73.**	(b)	**74.**	(a)	**75.**	(c)
76.	(b)	**77.**	(a)	**78.**	(b)	**79.**	(c)	**80.**	(b)
81.	(a)	**82.**	(b)	**83.**	(b)	**84.**	(b)	**85.**	(a)
86.	(c)	**87.**	(b)	**88.**	(d)	**89.**	(b)	**90.**	(a)
91.	(c)	**92.**	(a)	**93.**	(b)	**94.**	(d)	**95.**	(c)
96.	(b)	**97.**	(c)	**98.**	(a)	**99.**	(b)	**100.**	(d)
101.	(c)	**102.**	(a)	**103.**	(b)	**104.**	(c)	**105.**	(a)
106.	(a)	**107.**	(c)						